Physics I Semestral Examination, 2009, B.Math 1st Yr

Attempt all questions. Each question carries 10 marks. All results proved in class may be used by citing them, though results of exercises have to be proved in full. You may consult books and notes.

(1) In the presence of a force pointing towards the origin, a particle of unit mass describes a conic on the plane $\mathbb{R}^2$ given by the equation

$$\frac{l}{r} = 1 + e\cos\theta$$

in polar coordinates. Show that the force F must have magnitude $\frac{k}{r^2}$ for some constant k (i.e. must be an inverse square law force).

(2) Two billiard balls of masses m_1 and m_2 travel towards each other along the x-axis (which has unit basis vector $\widehat{\mathbf{e}}$) at velocities $u_1\widehat{\mathbf{e}}$ and $-u_2\widehat{\mathbf{e}}$ respectively, and collide head on. After the collision, they move with velocities $-v_1\widehat{\mathbf{e}}$ and $v_2\widehat{\mathbf{e}}$ respectively. Using conservation of energy and momentum, compute v_1 and v_2 in terms of u_1, u_2. (It might simplify the algebra to change to centre of mass coordinates).

$$\overset{\bullet}{m_1} \quad \overset{\longrightarrow}{u_1\widehat{\mathbf{e}}} \qquad \overset{\longleftarrow}{-u_2\widehat{\mathbf{e}}} \quad \overset{\bullet}{m_2}$$

(3) Let a rigid rod of uniform density, length l and mass m be suspended vertically from one end via a frictionless pivot, under a gravitational field. Let θ denote the angular displacement from the vertical. Write down a differential equation for θ, and solve it for small displacements θ. Thus determine the time period of oscillation.

(4) A circular revolving stage $\mathbb{F}_2$ rotates about a vertical axis passing though its centre with angular velocity $\omega\widehat{e}_3$ inside an auditorium $\mathbb{F}_1$ (which has basis $\{\widehat{e}_1, \widehat{e}_2, \widehat{e}_3\}$). A marble moves away with constant speed v away from the centre of the revolving stage radially outward in a direction fixed with respect to the stage $\mathbb{F}_2$. Find (i) its trajectory and (ii) its acceleration with respect to $\mathbb{F}_1$, assuming for simplicity that the centre of $\mathbb{F}_2$ is the origin of both $\mathbb{F}_2$ and $\mathbb{F}_1$ and fixed in time. Which component of the acceleration is Coriolis and which one is centrifugal ?

(5) A cylindrical drum of diameter D is kept in the gravitational field (with gravitational acceleration g). At the bottom of the drum is a drain-hole of diameter d, through which an ideal fluid flows out. If the initial height of the fluid is h, find out how long it takes for the drum to empty out.